Optimal Water Allocation in Shortage Situations as Applied to an Irrigation Community

Javier Alarcón ; Alberto Garrido , and Luis Juana, M.ASCE

Abstract: This work studies the most beneficial way of allocating water in an irrigation community in water shortage situations. Therefore, it proposes that the irrigation surface area be divided into homogeneous zones, each with a beneficial relationship with respect to the water applied. The mathematical formula that enables one to obtain the optimal quota for the users or irrigation community as a whole has been found for individual relations of a quadratic or power type, and these have yielded different and complementary characteristics. Dimensionless variables have been used to display the results, and to compare with other alternative allocation rules such as the proportional rule, referencing the situation without water restrictions. As a result, for each water shortage situation, the water that is allocated to each user is obtained, together with the losses in individual income and the losses for the community as a whole. Furthermore, a proposal is put forth for establishing the marginal benefit from the water available, which could be of interest in enabling each community to analyze whether it is in its best interest to invest in increasing the resource, or to sell the resource to other users. Finally, an example is given to demonstrate how the method works and to show that, when the differences between the production schemes are considered, the differences in benefit reduction between the proportional allocation and the optimal allocation are also sizeable.

Author keywords: Water shortages; Optimal allocation; Proportional allocation; Loss of income; Private benefit functions; Economic efficiency and equity.

Introduction

Irrigation generates a major part of the end product and employment in the agricultural sector, especially in regions where water resources are scarce. The Water Framework Directive [Official Journal of the European Union (OJ) 2000] constitutes a major boost to management practice, the purpose of which is to ensure that all EU water bodies are in a good state. Following the guidelines in the Water Framework Directive, the amended Water Act in Spain [Boletín Oficial del Estado (BOE) 2001] establishes that the general aims of hydrological planning are, for example, "to meet the demand, achieving a balance and harmonizing regional and sectorial development, economizing where the use of water is concerned and rationalizing its use in keeping with the requirements of the environment and the other natural resources." To achieve these objectives, the Hydrological Planning Regulations (BOE 2007b) consider a series of measures. However, this list of measures does not include the *reallocation of water use rights* in response, for example, to drought situations, although the Spanish Drought Management Plans passed in 2007 (BOE 2007a) developed the protocols for taking action and the measures that can be applied under these conditions.

When irrigating above the sustainable availability or renewable level at any given time, a series of environmental costs and opportunity costs, also referred to as resource costs, emerge, as explained in Alarcón et al. (2011). This state of affairs generally occurs when there is a period of water shortage or drought, and, in spite of this, the users continue to irrigate using the same amounts as they were before the shortage. In general, in Spain the allocation system is based largely on the order of priority of uses established in the legislation or in the planning regulations currently in force, and, to a lesser extent, on the agreements in drought committees and reservoir water release commissions. In irrigation communities, water is allocated proportionally, woody crops and those considered to be social often taking precedence, because they require more manpower (Calatrava and Garrido 2006). Reallocation or decentralization revolves around transactions involving water use rights (e.g., Garrido et al. 2012). Assigning rights is a basic question, not only in ensuring that the water markets operate smoothly, but also to enable the increasingly frequent periods of drought to be efficiently managed (Lorenzo-Lacruz et al. 2013). It is advisable to include additional economic criteria that are useful in improving efficiency, as well as fairness in the assignment of water rights.

Prices, markets, and quotas are the key tools for regulating water use, but if there is no suitable legal and institutional control, it is unlikely that any of these tools by themselves can bring about efficient water allocation (Tsur 2009). Regulation may either be direct, as through a system of pricing, quotas, or a combination of both; or indirect, with a water market that is more or less institutionally controlled. The possibilities vary depending on the physical characteristics (climate, soil, and water) and the economic, cultural, political, legal, and institutional characteristics of each particular case (Tsur 2009). The widely differing ways of allocating and charging for irrigation water in the world [see, e.g., Johansson et al. (2002) or Berbel et al. (2007)] thus reflect this variability. However, a priori, any water sharing or allocation could be achieved by imposing suitable quotas or a combination of quotas and pricing. Nevertheless, the results could differ greatly when these are applied, so the matter has to be studied carefully if the goal is to reduce environmental damage and keep the social cost to a minimum.

In this context, Goetz et al. (2005) simulate the introduction of three water allocation methods for irrigated land by comparing their economic efficiency: the proportional system, the market system, and the introduction of a uniform quota rule, developed under the theory of social choice. The results make it possible to conclude that, in theory, the water market leads to better overall results in all cases, particularly in severe water shortage situations. As an extension to their work, Goetz et al. (2005) refer to the development of nonanonymous allocation rules, i.e., rules that consider the differences in productivity among the various exploitations. On his behalf, Faysse (2003) makes a distinction between *ex-ante* and *ex-post* rules, depending on whether allocations are known before farmers' choices: An ex-ante allocation rule would be efficient if the group is homogeneous, whereas an ex-post rule would create more value. However, the ex-post allocation rule leads to large differences in farmers' profits. The best solution consists of linking it with a high water tariff, and ensuring that the taxes collected are redistributed to meet equity objectives. Finally, studies have been conducted by Syme and Nancarrow (1997) to establish whether social psychological theories of procedural and distributive justice could provide a basis for evaluating the "equity" or "fairness" of water allocation systems.

Optimization modeling has become popular in water management; it is primarily based on productive parameters. Irrigation water allocation among agricultural competing demands has been carried out with the aim of maximizing the overall economic benefits obtained, allocating available water to each user as a function of the water's profit margin (Babel et al. 2005; Benli and Kodal 2003; Letcher et al. 2004; Reca et al. 2001; Shangguan et al. 2002). Environmental release has also been considered to allocate water optimally in detrimentally irrigated agriculture, combining the drought cost on the environment and irrigation net profit in terms of water use (Grafton et al. 2011). However, in cases of water restrictions the sole target of maximizing the aggregated production in an irrigation area may inflict very different losses of income among farmers. In fact, constraints for equity in benefit maximization have been considered in Smout and Gorantiwar (2006) and in Gorantiwar and Smout (2007); the results of which show that the performance measures of productivity, equity, and adequacy conflict with each other.

Materials and Methods

Optimization models at a farm or irrigation district level must consider the cropping pattern in order to allocate water and land among alternative crops. In addition to water resources constraints and the stochastic nature of water availability, proportional reductions to initial allocations in response to water scarcity are often considered. One of the aims of this work is to provide guidance about the information needed to obtain the economically optimal water allocation; the focus is not only to establishing this idea on the basis of that information, but also to clearly show the cost of opportunity that other rules such as proportional allocation would have.

The theoretical basis of the optimal resource allocation among productive activities is known as Pareto optimal distribution. According to this principle, total benefits are maximized when levels of consumption are such that the marginal benefits for each use across all uses and time periods are equal (Reca et al. 2001). A similar reasoning could be considered when marginal losses instead of benefits may arise, namely, in water shortage situations.

Reallocation rules that take into account the actual irrigation can be sufficient to meet the standards of environmental quality, but they generally will not produce a Pareto-efficient allocation of water. The authors have found (unpublished results) that fixing the same final quota for every farmer would be efficient if the group is homogeneous (i.e., the same water productivity for all farmers). However, setting crop-specific quotas by eliminating any difference in farmers' losses would cause less damage to farmers as a whole if the group is heterogeneous (uneven water productivities). Even when imposing quotas implies fixing the same loss of income for every farmer, there may be very different effects in farmers' profit and loss, especially when there are substantial disparities in farmers' outputs, which is detrimental to the purpose of achieving equity in water allocation. In areas where there is little variety in crops and yields, applying the same loss of income (absolute) to all water users may be enough; however, when this is not the case, it is preferable to use relative loss of income, defined as a percentage of an initial income or reference income. This is the proposal being developed in this article.

Obtaining Water Benefit Functions

The information available indicates that it is advisable to complete and adapt when obtaining the benefit functions in connection with the water applied. The results provided by the proposed model would be more reliable if these functions are defined with greater accuracy for every crop or farm. In previous work in which the marginal returns on the crops are related to the amounts of water applied, it is customary to select a limited number of farms that are classified into homogenous groups of standard farms, on the basis of their response to water use. The modeling work on a farm level generally considers Leontief water-production relations, or takes the average yields from available time series. Another option is to construct yield functions taken from biophysical simulators of crop growth, as has been done by Goetz et al. (2005). Related to the latter option, AquaCrop (FAO, Rome) is the Food and Agricultural Organization of the United Nations (FAO) software that simulates the yield response to water at a given geographical location under climate change scenarios.

Explicit reference will be made to a simple way of determining the benefit functions in response to the water applied when a very limited amount of data is available, although it is preferable to obtain these functions through regression analysis, with a greater amount of data, to be more representative. More specifically, the net private marginal benefit function has to be established for every crop or farm in a particular irrigation zone, MB, which represents the marginal benefit or benefit variation on modifying the water applied in a unit, q, or, alternatively, its integral or net private benefit, B. Considering the benefit that can be obtained without irrigation, B_0, which could yield negative values, the authors can deduce this from the latter, defining a new function: $RB = B - B_0$. This function represents the additional net private benefit that is obtained by applying a water allocation q.

If restrictions have been imposed, the allocation for each water user, q, will be less than or equal to the reference allocation, without restrictions, q_r, which must be consistent with the crop requirements to obtain the maximum benefit. Thus, for q_r, the maximum benefit, RB_r, would be obtained, and this point (q_r, RB_r) will be taken as the reference point. Although other formulae could be used, for the purpose of simplicity, the quadratic benefit functions of the following type are used:

$$RB = B - B_0 = a \cdot q + \frac{m}{2} \cdot q^2, \qquad 0 \le q \le q_r, \qquad a > 0, \qquad m < 0 \tag{1}$$

$$MB = \frac{dB}{dq} = \frac{dRB}{dq} = a + m \cdot q \tag{2}$$

The expressions assume that it would be of interest to irrigate if the marginal benefit obtained from irrigation, MB, is greater than zero. Its value decreases as the amount of water applied increases. Therefore, the coefficient a has to be positive and the coefficient m must be negative. The double point condition, mathematical maximum, for the reference point (q_r, RB_r) indicates that the two parameters can be known only with this, and with it the functions

$$\left.\begin{array}{l} MB_r = 0 = a + m \cdot q_r \\ RB_r = a \cdot q_r + \frac{m}{2} \cdot q_r^2 \end{array}\right| \to a = \frac{2RB_r}{q_r}, \qquad m = -\frac{2RB_r}{q_r^2} \tag{3}$$

A ratio, q^*, can now be introduced, which expresses the fraction of the reference volume that is applied in water restriction situations: $q^* = q/q_r$. This relation will have its analogue for the increase in benefit obtainable with irrigation, RB^*, with respect to the reference situation. Operating with the value obtained for the two parameters a and m, under the mathematical maximum condition, Eqs. (1) and (2) are transformed into

$$RB^* = \frac{RB}{RB_r} = \frac{2q}{q_r} - \left(\frac{q}{q_r}\right)^2 = 2q^* - q^{*2} \tag{4}$$

$$MB^* = \frac{MB}{RB_r/q_r} = \frac{dRB^*}{dq^*} = 2 \cdot (1 - q^*) \tag{5}$$

Both are valid for $0 \le q^* = \frac{q}{q_r} \le 1$

It is also possible to obtain the reference benefit, RB_r, by knowing q_r and the benefit RB_a for any current allocation, $q_a < q_r$; by knowing $q_a^* = q_a/q_r$, in accordance with the first of the last two expressions, the following is obtained:

$$RB_r = \frac{RB_a}{2q_a^* \text{-} q_a^{*2}} \tag{6}$$

On occasion, reference will be made to reduction in benefit or loss of profit, L, when compared with the reference situation, which will be given by:

$$L = RB_r - RB = RB_r \cdot \left(1 - \frac{q}{q_r}\right)^2 \equiv L^* = \frac{L}{RB_r} = (1 - q^*)^2 \tag{7}$$

The dimensionless variables MB^*, RB^*, and L^* can follow a wide variety of functions and have coefficients or reference values that are significantly different; however, in their dimensionless form, they only depend on the variable q^*, and they are only represented in one way, as shown in Fig. 1.

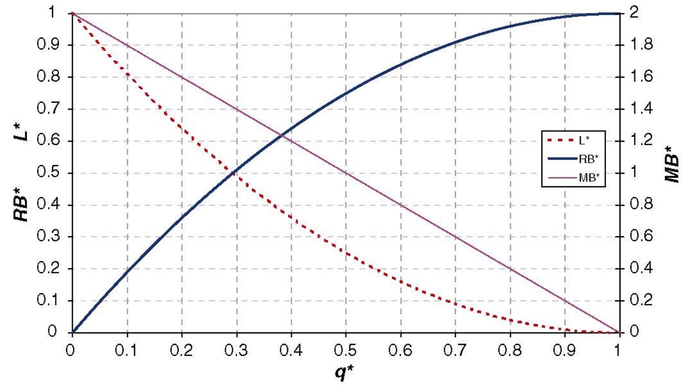

Fig. 1. Dimensionless quadratic benefit functions (RB^*, L^*, and MB^*) as a function of the allocation q^*

There may be crops or farms with different types of benefit functions RB within one single irrigation community. Furthermore, there could well be other sectors that are competing with irrigation for their own water requirements, and such sectors might have very different RB functions. In such cases, the power functions offer considerable versatility and could supplement the quadratic functions well. The following will hold for these:

$$RB = B - B_0 = c \cdot q^e, \qquad 0 \le q \le q_r, \qquad c > 0, \qquad 0 < e < 1 \tag{8}$$

$$MB = \frac{dB}{dq} = \frac{dRB}{dq} = c \cdot e \cdot q^{e-1} \tag{9}$$

Furthermore, it is assumed that it is of interest to irrigate if a marginal benefit MB greater than zero is obtained, and also that the marginal benefit decreases upon increasing the water applied. Therefore, the coefficient c and the exponent e will be greater than zero, and the latter will also be less than the unit. The independent term B_0 represents the benefit that is obtained when no amount of water is provided, and it could have negative values.

Two points will be necessary to determine functions RB_r and MB_r, but only one if the exponent e is known. In this study, one single point will be sufficient, which can logically be the reference point for the situation in which there is no water shortage. In the case of the power function, this point (q_r, RB_r) will not be the mathematical maximum. After e and the reference point are known, Eqs. (8) and (9) can be written as follows:

$$RB^* = \frac{RB}{RB_r} = q^{*e} \qquad MB^* = \frac{MB}{RB_r/q_r} = e \cdot q^{*e-1} \tag{10}$$

Both are valid for $0 \le q^* = \frac{q}{q_r} \le 1$

The loss of benefit that results from applying an amount $q < q_r$, i.e., from reducing the irrigation by a fraction $q^* < 1$, will be $L^* = \frac{L}{RB_r} = 1 - q^{*e}$.

In their dimensionless form, the three functions MB^*, RB^*, and L^* are singular for every exponent e. Fig. 2 shows the curves MB^* and RB^* associated with $e = 0.50$, by comparing the curves in Fig. 1.

Different exponents e can be used. The extreme value $e = 1$, function RB^* would be the bisector of the first quadrant; for the other extreme value, $e = 0$, it would be the horizontal line $RB^* = 1$.

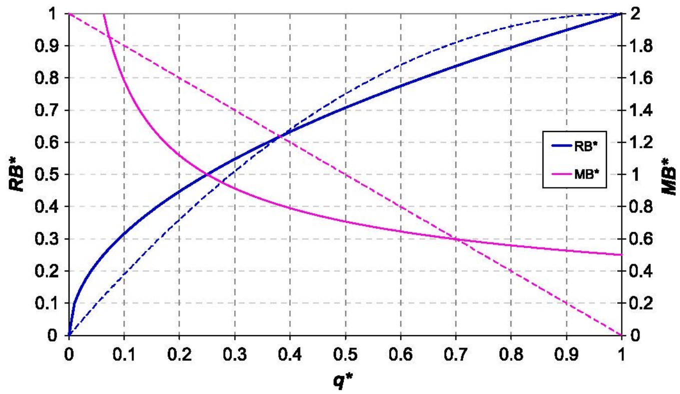

Fig. 2. Power dimensionless benefit functions (RB^* and MB^*) with exponent $e = 0.50$ (unbroken lines), compared with the quadratic functions (broken lines), on the basis of the allocation q^*

Model for Reallocating Water Quotas

Quadratic Functions

In an irrigation community, or in the set of productions for one single user with its respective functions defined by the pairs (q_{ri}, RB_{ri}), after the water sharing criterion has been established, a function is obtained that represents the benefit added to the water that is provided. In their dimensionless form, these functions for a set of production schemes or for the whole community, on the basis of the sharing criterion and the differences within the set, could be the same as or somewhat different from the individual functions that are represented in the preceding figures.

Henceforth, the same variables will be used to refer to the aggregated functions for a group or for an irrigation community, but using bold print, without italics, and with upper case letters. Thus, $\mathbf{Q}_R$ indicates the reference allocation for the whole community in conditions in which there is no water shortage and $\mathbf{Q}$ references the total allocation available at a given time; $\mathbf{RB_R}$, is the net benefit associated with the water, which the community would obtain when there is no water shortage, and $\mathbf{RB}$ corresponds to the allocation $\mathbf{Q}$. Logically, this latter aggregated benefit can have different values, depending on the water allocation that takes place. On the basis of the different users, the variables for the irrigation community would be

$$\mathbf{Q} = \sum q_i = \sum q_{ri} \cdot q_i^* \qquad \mathbf{Q_R} = \sum q_{ri}$$
$$\mathbf{RB} = \sum RB_i = \sum RB_{ri} \cdot (2q_i^* - q_i^{*2})$$
$$\mathbf{RB_R} = \sum RB_{ri} \qquad \mathbf{L} = \sum L_i = \sum RB_{ri} \cdot (1 - q_i^*)^2 \tag{11}$$

Proportional Allocation

The percentage reduction of water in a particular year, with respect to a year with no water shortage, $\mathbf{Q}^* = \mathbf{Q}/\mathbf{Q_R}$, allows for a proportional allocation that is the same for all of the water users: $q_i^* = \mathbf{Q}^*$. As a result, according to Eq. (11)

$$RB = RB_R \cdot (2Q^* - Q^{*2}) \tag{12}$$

$$L = RB_R \cdot (1 - Q^*)^2 \tag{13}$$

With this option, the loss of income, as a relative loss of benefit associated with the water, would be the same for all water users: $\mathbf{L}^* = L_i/RB_{ri} = (1 - \mathbf{Q}^*)^2$. For example, a value $\mathbf{Q}^*$ of 0.8 would enable an $\mathbf{L}^*$ of 0.04 to be obtained. This loss of income could be moderate, when compared with the water restriction, for the particularity of the maximum. However, if the frequent situation were that of an allocation such that $\mathbf{Q}^* = 0.8$, and a year in which $\mathbf{Q}^*$ was reduced to 0.60, this would indicate that the relative loss of benefit would go from 0.04 to 0.16. Therefore, the further the maximum allocation is stretched, the greater the economic losses.

Optimal Allocation

Alternatively, a more efficient water allocation can be sought, such that the total losses are minimized or that the aggregate benefit for the community is maximized. In this way, the q_i^* is established to make $\mathbf{L}$ minimum, with the limitation that the sum of the individual quotas allocated do not exceed the total amount of water available $\mathbf{Q}$, for the entire irrigation community.

In the optimal solution, the marginal benefit is the same for all those who receive an allocation, and the same as for the whole. Thus, for benefit functions of the quadratic type, the following would be obtained:

$$MB_i = \frac{RB_{ri}}{q_{ri}} \cdot 2 \cdot (1 - q_i^*) = \mathbf{MB} \rightarrow q_i^* = 1 - \frac{\mathbf{MB}}{2} \cdot \frac{q_{ri}}{RB_{ri}} \tag{14}$$

For the resulting value of $\mathbf{MB}$, as all assignations have to be greater than or equivalent to zero ($q_i > 0 \rightarrow q_i^* > 0$), the production schemes in which it does not hold that $2RB_{ri}/q_{ri} > \mathbf{MB}$ would not be included in the allocation. Therefore, when $\mathbf{MB}$ becomes known, the optimal allocation, incorporating logical variables that take unit value when they are true and zero value when they are false, would be obtained with

$$q_i^* = \left(1 - \frac{\mathbf{MB}}{2} \cdot \frac{q_{ri}}{RB_{ri}}\right) \cdot \left(\frac{2RB_{ri}}{q_{ri}} > \mathbf{MB}\right) \tag{15}$$

Therefore, every value of $\mathbf{MB}$ will correspond to an availability $\mathbf{Q}$, which can be determined by adding the assigned values

$$\mathbf{Q} = \sum q_{ri} \cdot q_i^* = \sum q_{ri} \cdot \left(1 - \frac{\mathbf{MB}}{2} \cdot \frac{q_{ri}}{RB_{ri}}\right) \cdot \left(\frac{2RB_{ri}}{q_{ri}} > \mathbf{MB}\right) \tag{16}$$

Determining this the opposite way, knowing $\mathbf{Q}$, determining $\mathbf{MB}$ must be done iteratively. Thus, on estimating a value, $\mathbf{MB_e}$, a value, $\mathbf{Q_e}$, is obtained with Eq. (6) that would be higher or lower than $\mathbf{Q}$. If it is higher, the estimation $\mathrm{MB_e}$ would have to be increased; if it is lower, the estimation would have to be decreased, until $\mathbf{Q_e} = \mathbf{Q}$.

Once $\mathbf{MB}$ is finally known from Eq. (16), and the water allocation is found with Eq. (15) and (11) can be used to obtain the community's benefits or losses. Furthermore, Eqs. (1) and (2) can be used to obtain those that are inherent to each production scheme.

The optimal solution is independent of the initial situation, regardless of whether it is optimal. That is to say, if there is a water shortage situation, $\mathbf{Q_A^*}$, with an allocation q_{ai}^* and a benefit $\mathbf{RB_A}$, which amounts to a known total loss $\mathbf{L_A}$, if the allocation is decreased or increased to $\mathbf{Q}^*$, the lower benefit loss or gain with respect to the previous situation leads to an allocation that is the same as if the reference situation were the initial situation.

Regarding the effect of the errors on establishing the parameters or reference points on the allocation, and assuming that those errors do not affect the value of $\mathbf{MB}$ established by the availability $\mathbf{Q}^*$, the following would be obtained:

$$\frac{dq}{dq_r} = 2q^* - 1 \rightarrow dq^* = \frac{dq}{q_r} = \frac{dq_r}{q_r} \cdot (2q^* - 1) \qquad (17)$$

$$\frac{dq}{dRB_r} = \frac{q_r}{RB_r} \cdot (1 - q^*) \rightarrow dq^* = \frac{dq}{q_r} = \frac{dRB_r}{RB_r} \cdot (1 - q^*) \qquad (18)$$

This would indicate that an error affecting q_r is positive, $dq_r > 0$, i.e., assuming that the requirements are greater than they actually are, it would mean receiving more water, $dq > 0$, if $q^* > 0.5$. However, the opposite could be true; it could indicate receiving less water, $dq < 0$, if $q^* < 0.5$, and the latter situation is more likely in severe water shortage conditions. For example, a positive error of 20% in $q_r(dq_r/q_r = 0.2))$ would lead to $q^* = 0.75$, an error $dq^* = 0.10$, thus leading to 10% extra water being received. By contrast, in a very dry year in which $q^* = 0.25$, the same error would lead to 10% less water being received.

However, a positive error affecting RB_r, $dRB_r > 0$, would lead to more water being received, $dq > 0$, whereas a negative error would mean receiving less water than the amount required if there were no errors. The effects of this error would be greater in years when the water shortage is more acute and for the production schemes in which there is less production; less water would be required. The same positive error of $\pm 20\%$, now in RB_r and where $q* = 0.75$, would indicate an error of $\pm 5\%$ in the allocation of water; however, the error would be $\pm 15\%$ if $q* = 0.25$.

Quadratic and Power Functions, or Functions of Another Type

In the case of power functions, the share, solving (10), would be

$$q_j^* = \left(\frac{\mathbf{MB}}{e} \cdot \frac{q_{rj}}{RB_{rj}}\right)^{1/(e-1)} \qquad (19)$$

In contrast to the quadratic functions, as far as the power functions are concerned, the users will never be entitled to a zero allocation, given that q_j^* will invariably be greater than zero. With the quadratic functions, no production scheme obtained the reference allocation in water shortage conditions. However, with the power functions, Eq. (19) could yield greater values than one, and these would logically be assigned the reference value $(q^* = 1)$. This will happen to all types that have a value of $e \cdot RB_r/q_r$ greater than the corresponding marginal benefit. That is to say

$$\mathbf{MB} \leq e \cdot \frac{RB_{rj}}{q_{rj}} \rightarrow q_j^* = 1 \qquad (20)$$

If logical variables are introduced, the allocation for a known value of **MB** could be expressed as follows:

$$q_j^* = \left(\frac{\mathbf{MB}}{e} \cdot \frac{q_{rj}}{RB_{rj}}\right)^{1/(e-1)} \cdot \left(\mathbf{MB} > e \cdot \frac{RB_{rj}}{q_{rj}}\right) + \left(\mathbf{MB} \leq e \cdot \frac{RB_{rj}}{q_{rj}}\right) \qquad (21)$$

If users compete with quadratic and power-type benefit functions, on equaling the marginal costs for obtaining the optimum, the following would be obtained:

$$MB_i = \frac{RB_{ri}}{q_{ri}} \cdot 2 \cdot (1 - q_i^*)^2 = MB_j = \frac{RB_{rj}}{q_{rj}} \cdot e \cdot q_j^{*e-1} = \mathbf{MB} \qquad (22)$$

Therefore, every MB has the right to an allocation, values q_i and q_j, or an availability **Q**, which can be calculated explicitly. As a result, after the availability **Q** is known, the value **MB** of the optimum can be calculated by solving the following implicit equation:

$$\mathbf{Q} = \sum q_{ri} \cdot \left(1 - \frac{\mathbf{MB}}{2} \cdot \frac{q_{ri}}{RB_{ri}}\right) \cdot \left(\frac{2RB_{ri}}{q_{ri}} > \mathbf{MB}\right) + \sum q_{rj} \cdot \left[\left(\frac{\mathbf{MB}}{e_j} \cdot \frac{q_{rj}}{RB_{rj}}\right)^{1/(e_j-1)} \left(\mathbf{MB} > e_j \cdot \frac{RB_{rj}}{q_{rj}}\right) + \left(\mathbf{MB} \leq e_j \cdot \frac{RB_{rj}}{q_{rj}}\right)\right] \rightarrow \mathbf{MB} \qquad (23)$$

Estimating a value of **MB**, using Eq. (23) enables one to calculate the value of **Q**. If this does not coincide with the quantity of water available, the estimation has to be modified by reducing **MB** (if **Q** is lower) or increasing it (if the opposite is the case), until it is the same.

Similarly, another type of function can be included in the model; Eq. (23) has been extended to power functions with various exponents e.

Case Study

The following example is an imaginary one applied to the model explained previously. It has been selected as a way to illustrate what a potential real case might look like. In an irrigation zone covering $\mathbf{A_R} = 9755$ ha, and on the basis of the crops, the production system, and the specific soil conditions, a distinction has been made among $n = 207$ types or production schemes in the farms as a whole. Each one of these types will have a representation on the surface, a_i (ha), for a particular year, and certain inherent or characteristic parameters, such as the allocations and reference benefits q_{ri} and RB_{ri}. The benefit function is intended to be studied for that particular year, for the entire irrigation community, **RB**, for the allocation concerning $\mathbf{Q} = 18.1$ hm^3, which is equivalent to 50% of the reference allocation or nonrestrictive allocation, considering an optimal share that reduces the aggregate loss of income to a minimum. Furthermore, the results are to be compared with those that would be obtained by applying a proportional share.

First, the water requirements were obtained for each crop, production system and location, using programs such as *CropWat* (FAO, Rome). The performance of the applications was estimated on the basis of the irrigation method and the irrigation programming and, thus, the gross requirements could be calculated as the reference allocations, q_r. Production estimates were also available, whether for the reference allocations q_{ri} and/or for other known allocations q_a, as well as certain expected sale prices and costs. As a result, certain benefits, B_r or B_a, were obtained that, together with the benefits that would be obtained for each crop if no irrigation were applied, B_0, enable the user to obtain the net reference benefits that can be attributed to irrigation RBr in each particular case. These must be checked and verified by qualified engineers, taking into consideration the experiences every year, and made available so that the predictions for each campaign can be made.

In this example, the reference allocation (without restriction) has an average value $q_{r\,\mathrm{med}} = \mathbf{Q_R}/\mathbf{A_R} = 3,711$ m^3 ha^{-1}, with certain major variations: a coefficient of variation $CV_q = 0.81$. The reference benefit resulting from irrigation has an average value $RB_{r\,\mathrm{med}} = \mathbf{RB}_R/\mathbf{A_R} = 2.005$ € ha^{-1}; its variations are also considerable: $CV_{RB} = 1.06$. The average value of the water productivity in the reference situation, RB_r/q_r, is $RB_r/q_{r\mathrm{med}} = \mathbf{RB}_R/\mathbf{Q_R} = 0.54$ € m^{-3} and the coefficient of variation is 1.24. The values below the average value for the three distributions have greater surface representation than the values above the average (Fig. 3).

Results and Discussion

Case of Quadratic Benefit Functions

The relation between the allocation available $\mathbf{Q}^* = \mathbf{Q}/\mathbf{Q_R} = q_{\mathrm{med}}/q_{r\,\mathrm{med}}$ and the marginal benefit $\mathbf{MB}$ that the irrigation community obtains with the optimal share can be calculated using Eq. (16). The result is shown in Fig. 4. In the case of the optimal share, when the allocation is low, a considerable percentage of the surface area could remain unirrigated; the so much per one of irrigated surface, $\mathbf{A}^* = \mathbf{A}/\mathbf{A}_R$, has also been shown.

As discussed previously, given that every $\mathbf{Q}^*$ has the right to an optimal share q_i, which is established using Eq. (15), with the respective value $\mathbf{MB}$, it is feasible to determine the benefit functions $\mathbf{RB}$ and $\mathbf{L}$ for the whole community. Both functions have been shown in dimensionless form, so they can be compared with the respective individual functions, all identical, which would also represent those for the community in a proportional allocation (Fig. 5).

In water shortage situations, the community receives a greater aggregate benefit from selective sharing, whose purpose is to grant more water to the most productive systems than if the allocations were reduced proportionally to the availability. The benefit $\mathbf{RB}$ for the example given with $\mathbf{Q}^* = 0.5$ is 90% of the maximum, compared with the 75% that would be obtained with proportional sharing (Fig. 5). Of the 207 types considered, 43 would be left without an allocation with the optimal share, which amounts to an unirrigated surface area of 1-$\mathbf{A}^* \cong 13\%$ (Fig. 4). The results for other values of $\mathbf{Q}*$ can be obtained in both graphs. The irrigation community can therefore have a clear and concise idea of the economic effects of the lack of water. Furthermore, the farmers can, individually and on the basis of the predictions, find out what quantities are allocated to each production scheme. Then they can plan their production accordingly.

Fig. 6 shows the distribution function $A*$ for individual relative allocations $q*$, for different water availabilities $\mathbf{Q}^*$.

In comparison with the proportional share, whereby with $Q* = 0.5$, all would receive $q*_i = 0.5$–, the optimal share demonstrates a major transfer of the least productive systems to the most

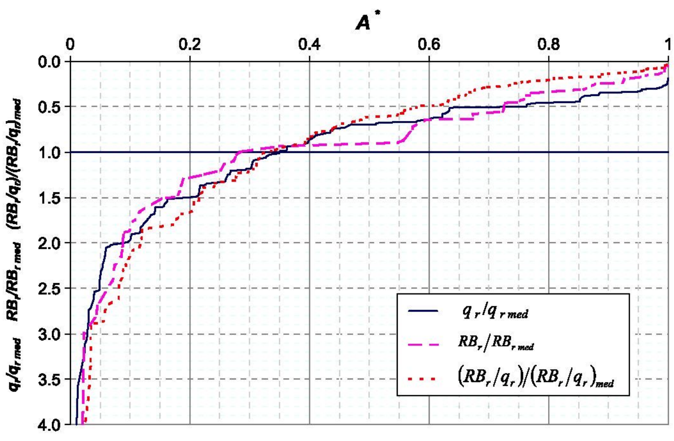

Fig. 3. Cumulative distribution function A^* for the unit reference values for water allocations, benefits, and productivities in the analyzed irrigation community

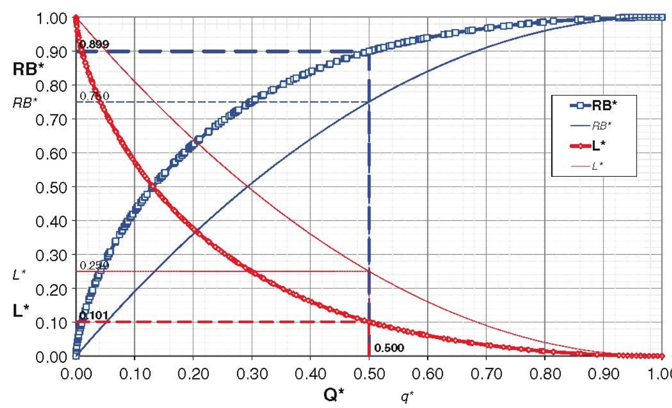

Fig. 5. Benefit functions RB^* and loss of income L^* for the whole community, on the basis of the relative allocation available Q^*, in the case of quadratic benefit functions

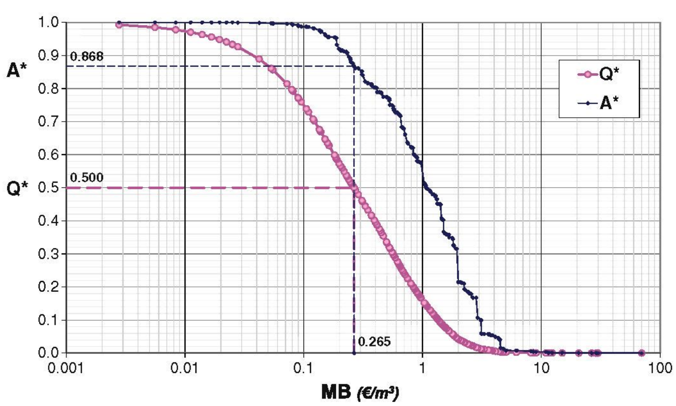

Fig. 4. Irrigation community's marginal benefit MB on the basis of the relative allocation available Q^*, with optimal share and surface under irrigation A^*, in the case of quadratic benefit functions

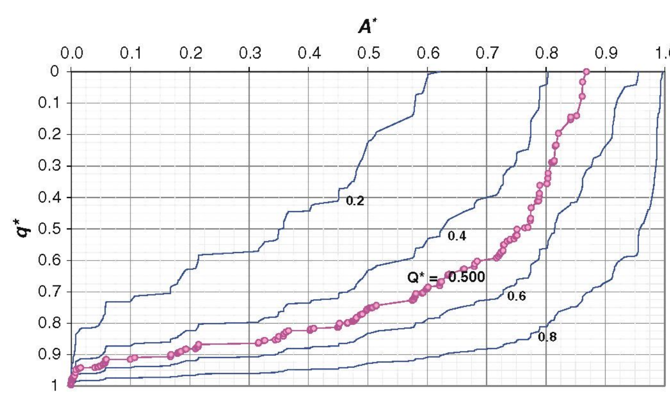

Fig. 6. Cumulative surface distribution function A^* for different allocations q^* and water availabilities Q^* (0.2, 0.4, 0.5, 0.6, and 0.8), in the case of quadratic benefit functions

productive systems: The lower that RB_r/q_r is, the lower relative allocation q^* it receives from the share. Thus, when $Q* = 0.5$ for the community being analyzed, approximately 70% of the surface receives more water than it would receive with the proportional share, $q*_i = 0.5$, at the expense of the other 30% of the surface area, which would be entitled to a smaller allocation than under proportional sharing and, above all, at the expense of 13% of the surface area, which would not receive any water. Differences between proportional and optimal allocations are reduced when the variability of the types is lower; however, there is considerable variability in the example studied.

Case of Power Benefit Functions

What would happen if the benefit functions were power ones, with an exponent $e = 0.5$, and they passed through the reference point (q_{ri}, RB_{ri}), obtained and used in the quadratic functions? The different shape of these curves (Fig. 2) shows that no water user is left without an allocation, and that, by contrast, there is one group of production schemes that receives the same amount as it would receive without a water shortage. The relation between the relative allocation available and the resulting marginal benefit is shown in Fig. 7.

Furthermore, the benefit and loss of income unit functions, on the basis of the allocation available, are shown in Fig. 8. It is possible to compare these functions in their dimensionless form,

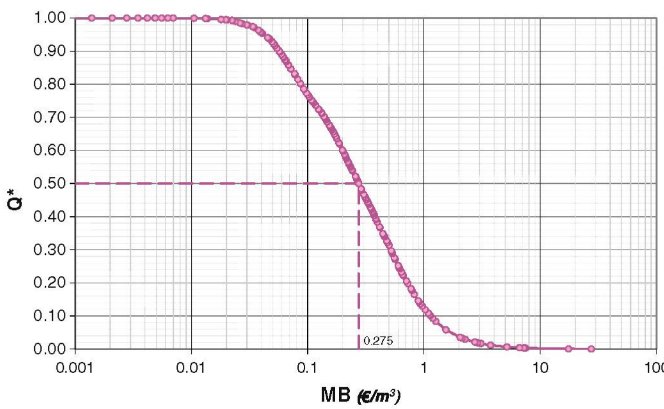

Fig. 7. Marginal benefit MB for the community, on the basis of the relative allocation available Q^* with optimum share, in the case of power benefit functions

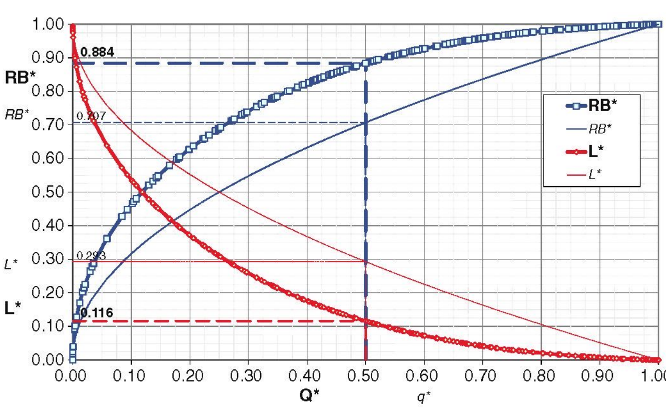

Fig. 8. Benefit functions RB^* and loss of income L^* for the whole community, on the basis of the relative allocation available Q^*, in the case of power benefit functions

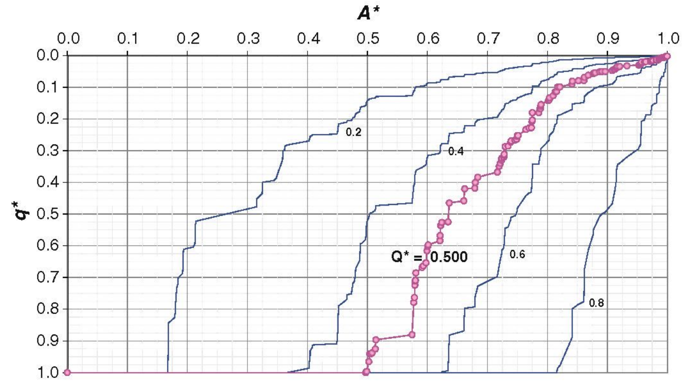

Fig. 9. Cumulative surface distribution function A^* for different allocations q^* and water availabilities Q^* (0.2, 0.4, 0.5, 0.6, and 0.8), in the case of power benefit functions

with the respective individual functions, all identical, which would also represent (in this case with one single exponent e) those for the community with a proportional share. Once again, in water shortage situations, the community obtains a significantly greater aggregate benefit with the optimal share than with the proportional rule. The lower the variability for the types differentiated, the slighter the differences between the proportional share and the optimal share. However, in contrast to the previous case, now no type is left without water (Fig. 9).

Finally, the distribution or share for the unit allocations in each type, with respect to the percentage of surface area under irrigation, and according to the various water availabilities $\mathbf{Q}^*$, is shown in Fig. 9. A different curvature is found in these functions, compared with those obtained previously, for quadratic functions.

When compared with the proportional share (where $\mathbf{Q}^* = 0.5$, all would receive $q*_i = 0.5$), the optimal share demonstrates a major transfer of the less productive systems to the more productive systems. Thus, when $\mathbf{Q}^* = 0.5$ for the community analyzed, approximately 63% of the surface area receives more water than the amount it would be entitled to with the proportional share, $q*_i = 0.5$, to the detriment of 37% of the surface area to which a smaller amount would be allocated than with the proportional share. Almost 50% of the surface area would not receive an allocation reduction, indicating that the least productive types have a water consumption per unit of surface area that is considerably greater than the most productive ones.

Conclusions

This work presents a method for establishing the allocation of water quotas with the maximum benefit criterion, which could be more useful than the traditional proportional method or imposing single and indiscriminate allocations, when the heterogeneous nature of crop growing is a major factor. This optimal allocation method ensures that an irrigation community would incur certain opportunity costs that are lower than what they would be if any of those rules were applied. It even offers improvements with respect to the specific allocation assignment rule for crops and yields, based on the comparison of absolute losses of income among heterogeneous farmers, because these very different losses would be reduced to a minimum.

The use of dimensionless variables enables one to show how an irrigation community reacts economically to changes of water availability, and thus makes it possible to synthesize this economic behavior, all of which could be of interest when it comes to managing the availability of water. Although the numerical application is very specific, it serves to illustrate the method and its potential use and effects. This method could be applied generally to any river basin or hydrological system where there are trade-offs among water diversions (agricultural, industrial, or domestic) and even better where environmental flows are coupled with adequate data on flows and benefits of alternative uses.

An irrigation community that implemented this method could adjust the reduction of allocations more reliably, adapting them to each production scheme, while at the same time suggesting which plots could be left unirrigated, because of their low profitability. Such information could be useful in enabling farmers to plan their farming and irrigation efficiently, by achieving a lower aggregated cost. The same proposed method could also be used by a single user in deciding what to do when it comes to applying water to his or her various production schemes.

In addition, by knowing the reductions in irrigation diversions that would come from ceasing low-profit activities, the incremental environmental flows could be estimated. Therefore, it would be useful to implement all of this information, even if it had to be improved and updated every year, because it would enable irrigators and water planners to better evaluate the cost of the water shortage and, thus, analyze whether it might be worth looking into alternative solutions. If that information and the proposed method enable farmers to be more competitive, its use should be promoted.

Furthermore, to achieve an economically optimal water allocation for a community, a method that is controlled and reached by consensus should be established for obtaining the information and the basic parameters; this will reduce the risk of potential conflicts among users who might feel that they are being less favored or more discriminated against than others. As a result, water users would be able to understand that if their losses are lower when they are united, it might be in their interest to work together, establishing their respective compensations, even if it gives rise to certain difficulties.

After the reassignment of water use rights has taken place in a drought situation, introducing this market idea might further enhance economic efficiency in the use of irrigation water. In this sense, marginal benefit curves as obtained in this paper and, in particular, the resulting loss of income value in each case, can serve as reference to establishing the price of water in the transactions or exchanges among the users, as well as for studying the potential cost of increasing the water resources.

Finally, if the tracking of the crops that are sown is sufficient, applying the proposed rule should not cause any technical difficulties. If the crop type and its yield affect the amount of water that irrigators receive in a water shortage situation, it is possible that they would decide to change them. However, and in accordance with the allocation rule structure, it is also likely that they would try to improve their efficiency. Hence, further in-depth research is required, because it would offer sound theoretical properties.

References

Alarcón, J., Mesa-Jurado, M. A., and Berbel, J. (2011). "Design of irrigation water tariffs with overconsumption sanctions." *Spanish J. Agric. Res.*, 9(4), 971–980.

AquaCrop [Computer software]. Food and Agricultural Organization of the United Nations (FAO), Rome.

Babel, M. S., Das Gupta, A., Nayak, D. K. (2005). "A model for optimal allocation of water to competing demands." *Water Resour. Manage.*, 19(6), 693–712.

Benli, B., and Kodal, S. (2003). "A non-linear model for farm optimization with adequate and limited water supplies: Application to the South-East Anatolian Project (GAP) Region." *Agric. Water Manage.*, 62(3), 187–203.

Berbel, J., Calatrava, J., and Garrido, A. (2007). "Water pricing and irrigation: A review of the European experience." Chapter 13, *Irrigation water pricing policy in context: Exploring the gap between theory and practice*, F. Molle, J. J. Berkkoff, and R. Barker, eds., CABI, Wallingford, U.K., 295–327.

Boletín Oficial del Estado (BOE). (2001). Real Decreto Legislativo 1/2001, of July 20, that gives approval to the compiled text of the Water Law (Texto refundido de la Ley de Aguas), No. 176, July 24, 2001 (in Spanish).

Boletín Oficial del Estado (BOE). (2007a). Orden MAM/698/2007, of March 21, that gives approval to the special plans of action in alert and potential drought situations regarding hydrological plans in interregional river basins, No. 71, March 23, 2007 (in Spanish).

Boletín Oficial del Estado (BOE). (2007b). Real Decreto 907/2007, of July 6, that gives approval to the Water Planning Regulations [Reglamento de la Planificación Hidrológica], No. 162, July 7, 2007 (in Spanish).

Calatrava, J., and Garrido, A. (2006). "Difficulties in adopting formal water trading rules within user's associations." *J. Econ. Issues*, 40(1), 27–44.

CropWat [Computer software]. Food and Agricultural Organization of the United Nations (FAO), Rome.

Faysse, N. (2003). "Allocating irrigation water: The impact of strategic interactions on the efficiency of rules." *Eur. Rev. Agric. Econ.*, 30(3), 305–332.

Garrido, A., Rey, D., and Calatrava, J. (2012). "Water trading in Spain." Chapter 16, *Water, agriculture and the environment in Spain: Can we square the circle?*, L. De Stefano and M. R. Llamas, eds., Taylor and Francis, London, 205–216.

Goetz, R. U., Martínez, Y., and Rodrigo, J. (2005). "Eficiencia de las reglas de asignación de agua en el regadío: Asignación a través de mercados, de la regla proporcional y de la regla uniforme." *Economía Agraria y Recursos Naturales*, 5(9), 115–138 (in Spanish).

Gorantiwar, S. D., and Smout, K. (2007). "Model for performance based land area and water allocation within irrigation schemes." *Irrig. Drain. Syst.*, 20(4), 345–360.

Grafton, R. Q., Chu, H. L., Stewardson, M., and Kompas, T. (2011). "Optimal dynamic water allocation: Irrigation extractions and environmental tradeoffs in the Murray River, Australia." *Water Resour. Res.*, 47(12), W00G08.

Johansson, R. C., Tsur, Y., Roe, T. L., Doukkali, R. M., and Dinar, A. (2002). "Pricing and allocation of irrigation water: A review of theory and practice." *Water Policy*, 4, 173–199.

Letcher, R. A., Jakeman, A. J., and Croke, B. F. W. (2004). "Model development for integrated assessment of water allocation options." *Water Resour. Res.*, 40(5), W05502.

Lorenzo-Lacruz, J., Morán-Tejeda, E., Vicente-Serrano, S. M., and López-Moreno, J. I. (2013). "Streamflow droughts in the Iberian Peninsula between 1945 and 2005: Spatial and temporal patterns." *Hydrol. Earth Syst. Sci.*, 17(1), 119–134.

Official Journal of the European Union (OJ). (2000). Directive 2000/60/EC of the European Parliament and of the council establishing a framework for the community action in the field of water policy, L 327, December 22, 2000.

Reca, J., Roldan, J., Alcaide, M., Lopez, R., and Camacho, E. (2001). "Optimisation model for water allocation in deficit irrigation systems: I. Description of the model." *Agric. Water Manage.*, 48(2), 103–116.

Shangguan, Z., Shao, M., Horton, R., Lei, T., Qin, L., and Ma, J. (2002). "A model for regional optimal allocation of irrigation water resources under deficit irrigation and its applications." *Agric. Water Manage.*, 52(2), 139–154.

Smout, K., and Gorantiwar, S. D. (2006). "Productivity and equity of different irrigation schedules under limited water supply." *J. Irrig. Drain. Eng.*, 132(4), 349–358.

Syme, G. J., and Nancarrow, B. E. (1997). "The determinants of perceptions of fairness in the allocation of water to multiple uses." *Water Resour. Res.*, 33(9), 2143–2152.

Tsur, Y. (2009). "On the economics of water allocation and pricing." *Ann. Rev. Resour. Econ.*, 1, 513–536.